Unbox Model-based Reconstruction: Examples Employing 7T Diffusion MRI

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Introduction

Model-based reconstruction [1,2] involves two ingredients: (1) a nonlinear forward model, and (2) the minimization of the nonlinear inverse problem with regularization to solve for the corresponding model parameters. Thus, its implementation requires the construction of nonlinear operators and their corresponding Jacobian matrices during the minimization procedure. It becomes even more complicated when advanced regularization (e.g. ℓ^1 soft-thresholding) is employed.

To address these challengens, we built a generalized nonlinear operator framework in SigPy, stemming from its object-oriented linear operator abstraction [3]. Further, we implemented a general nonlinear inversion solver that alternates between model parameters update and advanced regularization terms.

Contribution 1: Nonlinear Operator Abstraction

For instance, diffusion tensor imaging (DTI) [4] presents a nonlinear operator:

$$\mathbf{E} : x = (b_0, D)^T \mapsto y = b_0 \cdot \exp(B \times D) \tag{1}$$

where b_0 and D are the non-diffusion-weighted image and the diffusion tensor, respectively. B is the diffusion encoding matrix. y are the diffusion-weighted images. In Python, such an nonlinear operator is constructed with the following class:

```
class Nlop():
    def _forward(self, x):          # compute the forward model output
    def _get_Jacobian(self, x):     # compute the forward model's Jacobian matrix
    def _derivative(self, x, dx):   # evaluate derivative
    def _adjoint(self, x, dy):      # evaluate conjugate transpose of derivative
```

Further, we provided a child class "Compose" to allow for the chain between nonlinear operators and linear operators (such as the parallel imaging model). Therefore, the complete nonlinear forward model in diffusion tensor imaging is,

$$\mathbf{A} : \mathbf{M}\boldsymbol{\Sigma}\mathcal{F}\mathbf{S}\boldsymbol{\Phi}\mathbf{E} \tag{2}$$

which presents diffusion tensor imaging with multi-band multi-coil acquisition. Specifically, diffusion-weighted images computed from $\mathbf{E}$ is multiplied by shot-to-shot phase variation $\boldsymbol{\Phi}$ and then by coil sensitivity maps $\mathbf{S}$. Note that $\mathbf{S}$ is FOV shifted in accordance with CAIPIRINHA [5]. The multi-slice multi-coil diffusion-weighted images are then Fourier transformed $\mathcal{F}$ and collapsed in the slice dimension $\boldsymbol{\Sigma}$. Finally, the collapsed k-space is masked by sampling pattern $\mathbf{M}$.

Contribution 2: Nonlinear Least Square Solver

Nonlinear inverse reconstruction of diffusion tensor model parameters x in eq. (1) reads,

$$\underset{x}{\operatorname{argmin}} \, \|y - \mathbf{A}(x)\|_2^2 + \lambda R(Tx) \tag{3}$$

where y is the measured k-space data. $R(x)$ is the regularization on transformed Tx with regularization strength λ. Here, we proposed to split the nonlinear least square part and the regularization part using the alternating direction method of multipliers (ADMM) [6],

$$\begin{aligned} x^{(k+1)} &:= \underset{x}{\operatorname{argmin}} \left\| y - \mathbf{A}(x^{(k)}) \right\|_2^2 + \rho/2 \left\| Tx^{(k)} - z^{(k)} + u^{(k)} \right\|_2^2 \\ z^{(k+1)} &:= \mathcal{T}_{\lambda/\rho}(Tx^{(k+1)} + u^{(k)}) \\ u^{(k+1)} &:= u^{(k)} + Tx^{(k+1)} - z^{(k+1)} \end{aligned} \tag{4}$$

x update is solved with the iteratively regularized Gauss-Newton method, whereas z update is solved with the proximal operator as singular value thresholding [7].

Methods

In vivo brain diffusion MRI with 2-shot interleaved echo planar imaging (EPI) was conducted at $7\,\mathrm{T}$ (Terra, Siemens Healthineers, Erlangen, Germany) with 32-channel receive coils. Acquisition parameters were $1.2\,\mathrm{mm}$ isotropic resolution with 94 slices, 3-fold in-plane acceleration, multi-band factor 2, and total scan time of $5\,\mathrm{min}$. 30 diffusion directions with b-value $1000\,\mathrm{s/mm^2}$ and 2 directions with b-value $50\,\mathrm{s/mm^2}$.

To solve eq. (3), the unknowns b_0 and D were initialized with 10^{-4} and 0, respectively. $\lambda = 10^{-6}$ and $\rho = 10^{-3}$. x was updated with 4 Gauss-Newton steps, and a total of 6 ADMM iterations were employed.

Results

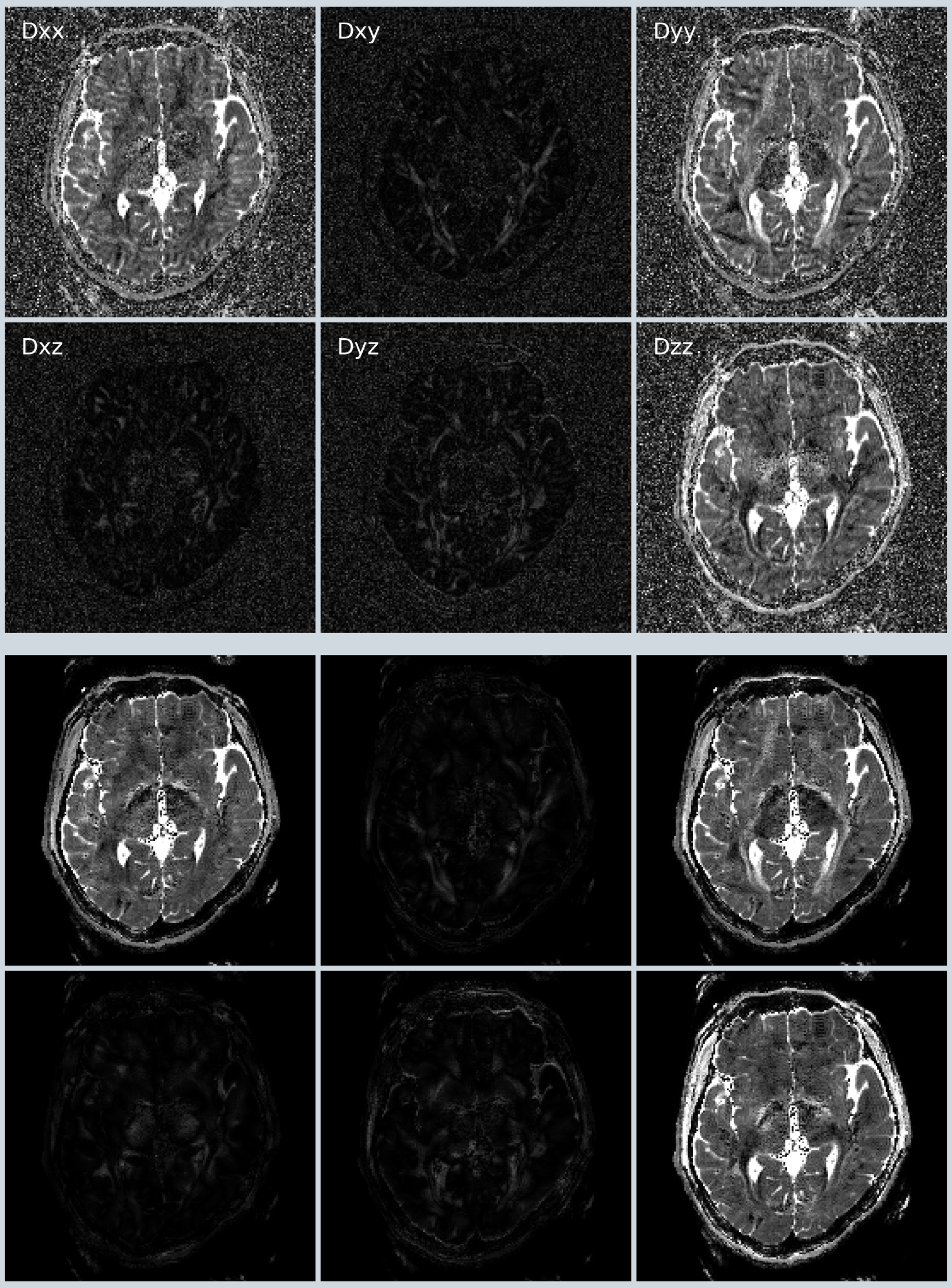

Fig. 1: Diffusion tensor maps reconstructed by (top) parallel imaging with pixel-wise model fitting and (bottom) model-based reconstruction. The model-based reconstruction shows reduced noise in the off-diagonal tenor maps.

Discussion & Conclusion

- Nonlinear operator abstraction in SigPy;
- Nonlinear least square solver with advanced regularization terms;
- Examples in $7\,\mathrm{T}$ diffusion MRI show reasonable results compared to parallel imaging with pixelwise model fitting;
- This framework may allow for fast prototyping and testing.

References

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