Free-Breathing Water, Fat, R_2^* and B_0 Field Mapping of the Liver Using Multi-Echo Radial FLASH and Regularized Model-based Reconstruction

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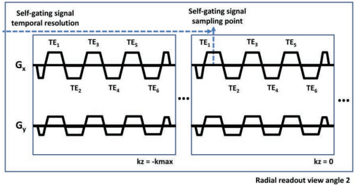
September 09, 2021

Motivation: Free-Breathing Liver Fat and R_2^* Mapping

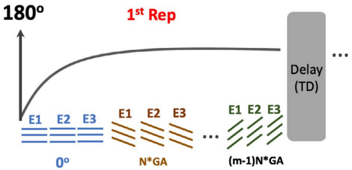
- multi-echo bipolar readout[1]
- self gating
- parameter fitting



- model-based reconstruction[2]
- calibrated coil sensitivity and B_0 maps

$$\operatorname{argmin}_{\mathbf{W},\mathbf{F},\mathbf{R}_2^*} \sum_{c,t_n} \| E(\mathbf{W},\mathbf{F},\mathbf{R}_2^*)_{c,t_n} - \mathbf{Y}_{c,t_n} \|_2^2 + \lambda_W \| S(\mathbf{W}) \|_1 + \lambda_F \| S(\mathbf{F}) \|_1 + \lambda_{R_2^*} \| S(\mathbf{R}_2^*) \|_1 .$$

- 3-point Dixon
- subspace reconstruction
- water-specific T1 mapping[3]

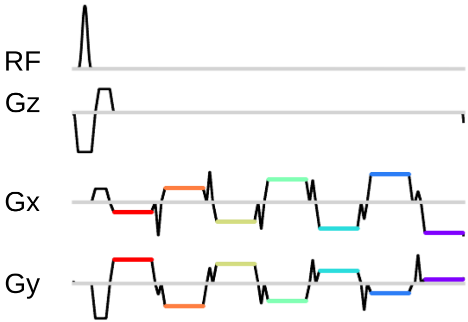


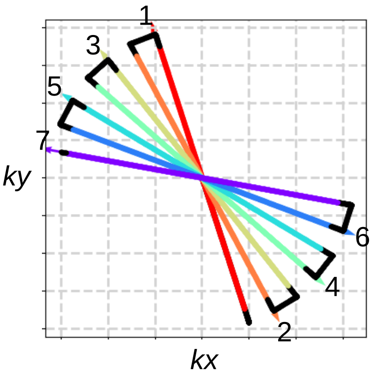
[1] Zhong X, et al. Effect of respiratory motion on free-breathing 3D stack-of-radial liver R_2^* relaxometry and improved quantification accuracy using self-gating. MRM (2019)

[2] Schneider M, et al. Free-breathing fat and R_2^* quantification in the liver using a stack-of-stars multi-echo acquisition with respiratory-resolved model-based reconstruction. MRM (2020)

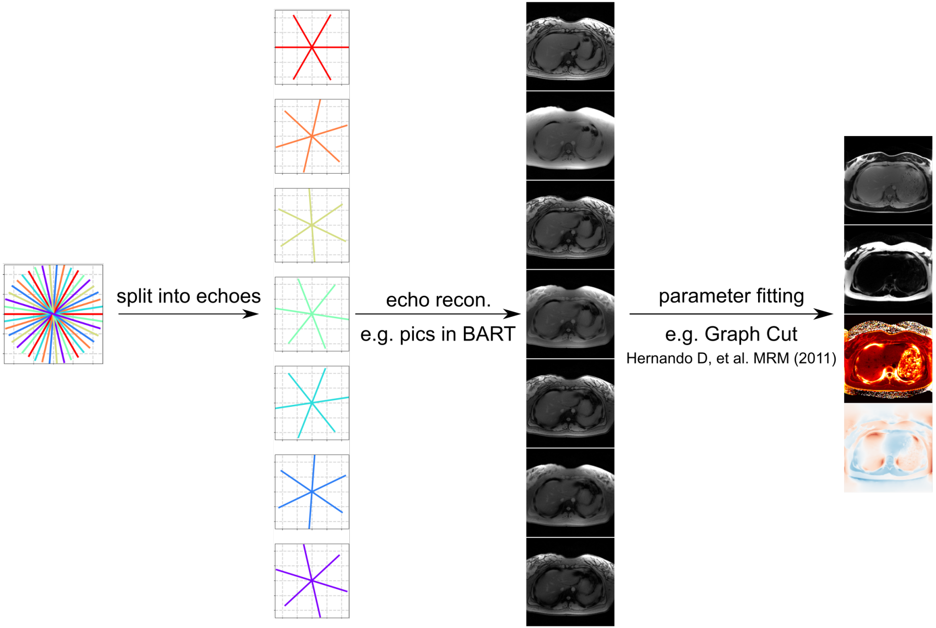
[3] Feng L, et al. Magnetization-prepared GRASP MRI for rapid 3D T1 mapping and fat/water-separated T1 mapping. MRM (2021)

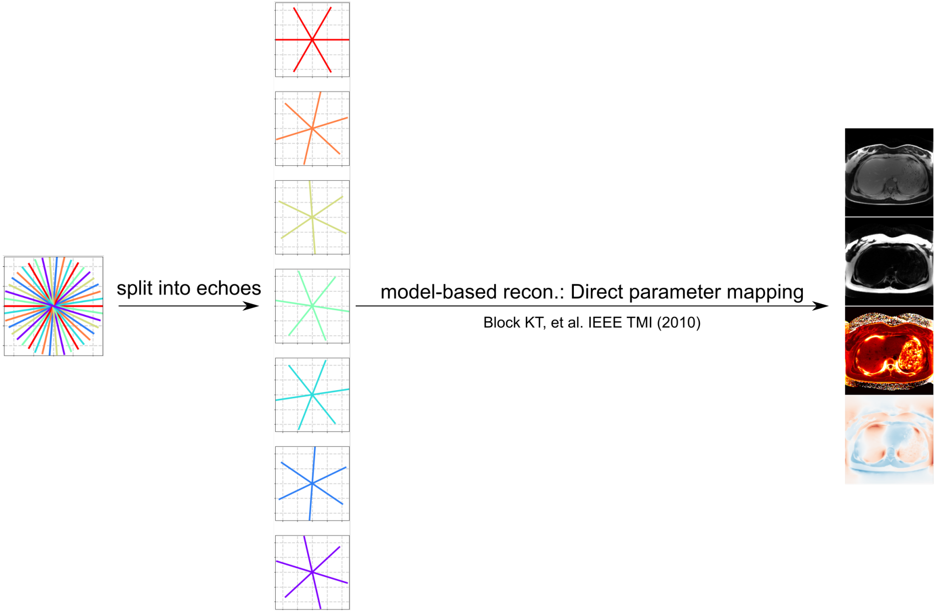
Multi-Echo Radial FLASH Sampling





- Seven gradient echoes with different *k*-space spokes per RF excitation
- Echo acquisition (ADC on) is color-coded


split into echoes
echo recon.
e.g. pics in BART
parameter fitting
e.g. Graph Cut
Hernando D, et al. MRM (2011)


split into echoes
model-based recon.: Direct parameter mapping
Block KT, et al. IEEE TMI (2010)

Multi-Echo Gradient-Echo Signal Modeling

- Chemical-shift and B_0 field inhomogeneity encoding (WFR2S in BART[1])

$$\mathcal{B} : x_p \mapsto \rho_m = (\mathrm{W} + \mathrm{F} \cdot z_m) \cdot e^{-R_2^* \mathrm{TE}_m} \cdot e^{i2\pi f_{B_0} \mathrm{TE}_m}$$

  - 6-peak fat spectrum[2]: $z_m = \sum_i \alpha_i e^{i2\pi f_i \mathrm{TE}_m}$
  - Inclusion of R_2^* extends the Dixon model[3,4]

- Given $x = (\underbrace{\mathrm{W}, \mathrm{F}, R_2^*, f_{B_0}}_{x_p}, \underbrace{c_1, \cdots, c_N}_{\text{coil sensitivities}})^T$, denote the chained forward model

$$y_{j,m} = F_{j,m}(x) := P_m \mathcal{F} S \mathcal{B}$$

[1] Uecker M, et al. Berkeley Advanced Reconstruction Toolbox. ISMRM (2015)

[2] Hamilton G, et al. In vivo characterization of the liver fat 1H MR spectrum. NMR Biomed (2011)

[3] Dixon WT. Simple proton spectroscopic imaging. Radiology (1984)

[4] Glover GH. Multipoint Dixon techniques for water and fat proton and susceptibility imaging. JMRI (1991)

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Regularized Model-based Iterative Reconstruction

$$\text{minimize}_x \; \|y - F(x)\|_2^2 + \lambda_1 \|\mathcal{W}(E_1 x)\|_1 + \lambda_2 \|\text{TV}_t(x_p)\|_1 + \lambda_3 \|x\|_2^2$$
$$\text{subject to } R_2^* \geq 0$$

- Spatial ℓ^1-Wavelet regularization on W, F, and R_2^* maps extracted by E_1
- Spatial & Temporal total variation (TV) regularization on x_p
- ℓ^2 regularization on all unknowns

Regularized Model-based Iterative Reconstruction

- Nonlinear inversion: IRGNM[5] with ADMM[6]
  1. Linearize the nonlinear objective function in each Newton iteration
  2. Solve the linearized problem with regularization via ADMM

- Initialization
  1. Newton-type method sensitive to initial guess, especially for B_0 estimate
  2. Initialize W, F and f_{B_0} using simplified model-based reconstruction from the first three echoes[7]

[5] Bakushinsky AB, et al. Iterative methods for approximate solution of inverse problems. Mathematics and Its Applications (2004)

[6] Boyd S, et al. Distributed optimization and statistical learning via the alternating direction method of multipliers. Foundations and Trends in Maching Learning (2010)

[7] Tan Z, et al. Dynamic water/fat separation and B_0 inhomogeneity mapping. MRM (2019)

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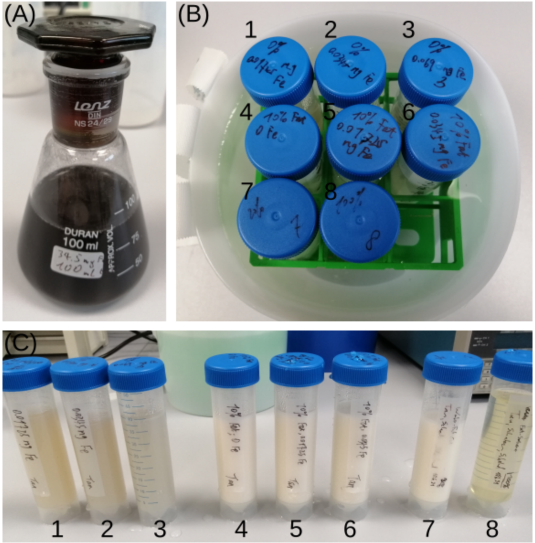
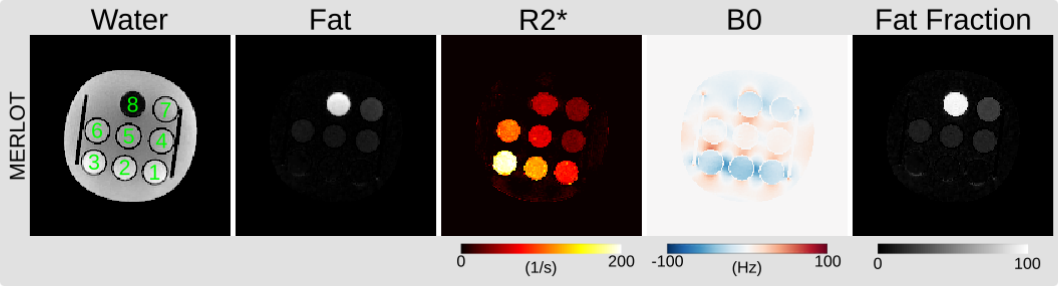
Fat & Iron Phantom



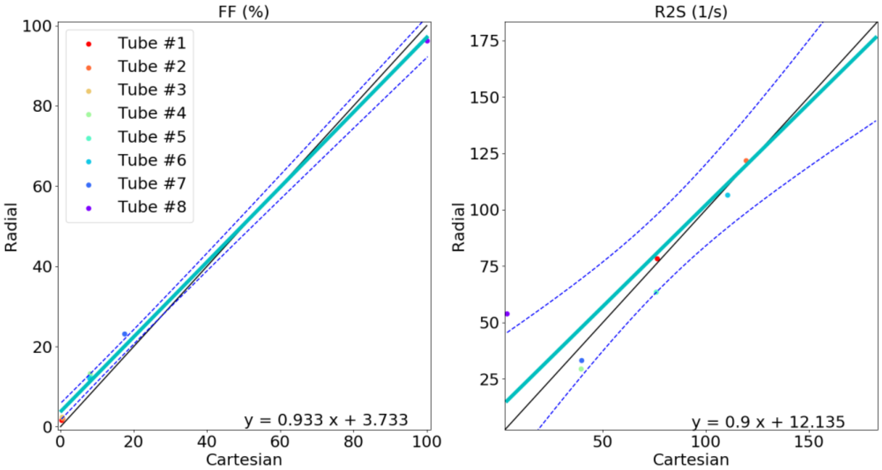
(A) Iron diluted in water
(B) Phantom outline
(C) Tubes

- 1 – 3: No fat, varying iron
- 4 – 6: 10 % fat, varying iron
- 7: 20 % fat, no iron
- 8: 100 % fat, no iron

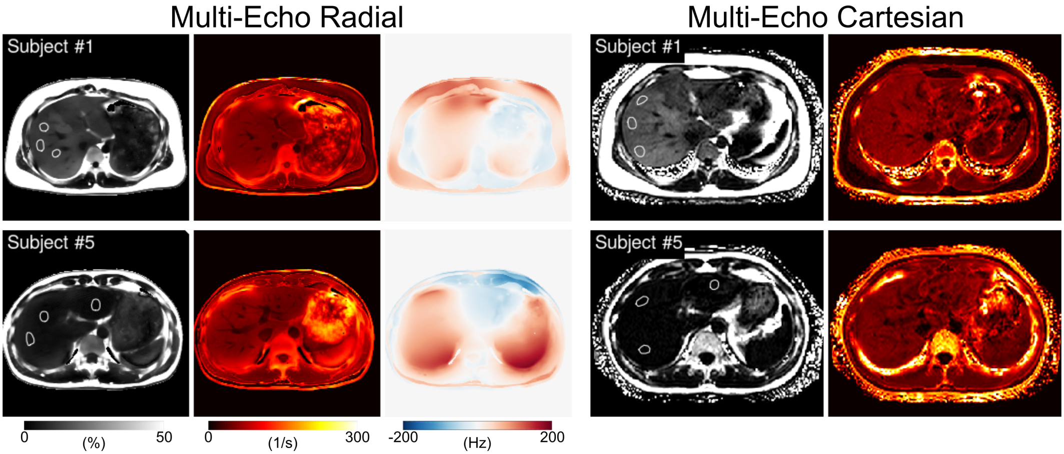
Results: Fat & Iron Phantom



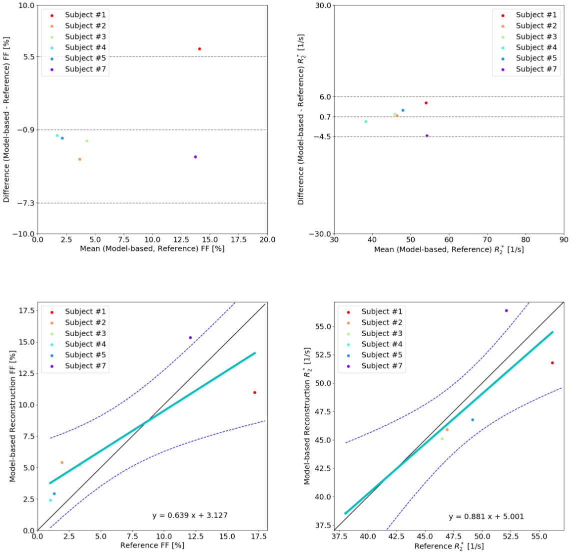
Results: Fat & Iron Phantom



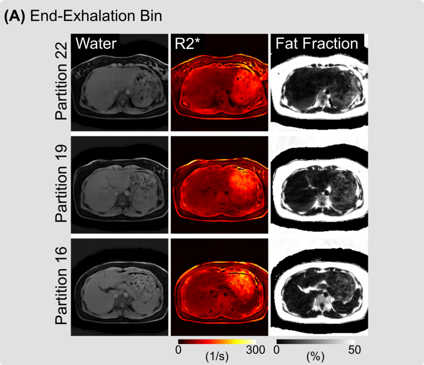
Results: Elevated Fat Fraction in the Liver

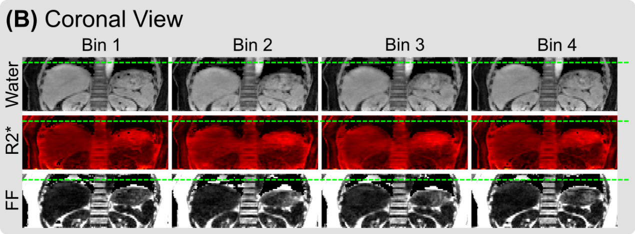


Results: Quantitative Analysis



Results: 3D Free-Breathing Acquisition in Two Minutes





- stack-of-radial multi-echo acquisition
- SSA-FARY[7] self gating
- multi-dimensional model-based recon.

[7] Rosenzweig S, et al. Cardiac and Respiratory Self-Gating in Radial MRI Using an Adapted Singular Spectrum Analysis (SSA-FARY). IEEE TMI (2020)

Summary

- Free-breathing liver acquisition via multi-echo radial FLASH
- Multi-gradient-echo model-based recon. with advanced regularization in BART
- Good quantitative accuracy comparing with the reference method
- Associated reconstruction scripts will be updated here:
  `https://github.com/mrirecon/multi-echo-liver`

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Iteratively Regularized Gauss-Newton Method (IRGNM)

- IRGNM linearizes the nonlinear forward model in every Newton step,

$$\|F(x_n) + DF(x_n)\mathrm{d}x - y\|_2^2$$

- denote $x = x_n + \mathrm{d}x - x_0$,

$$\begin{aligned} &\|DF(x_n)(x + x_0 - x_n) - [y - F(x_n)]\|_2^2 \\ \Rightarrow &\|DF(x_n)x - [DF(x_n)(x_n - x_0) + y - F(x_n)]\|_2^2 \end{aligned}$$

- its normal equation reads,

$$\begin{aligned} &DF^H(x_n)\{DF(x_n)x - [DF(x_n)(x_n - x_0) + y - F(x_n)]\} = 0 \\ \Rightarrow &DF^H(x_n)DF(x_n)x = DF^H(x_n)\{DF(x_n)(x_n - x_0) + y - F(x_n)\} \end{aligned}$$

- with arbitrary regularization, ADMM can be used to solve it efficiently.

Generalized $\ell 1$ Regularization via ADMM

$$\begin{aligned} \text{minimize} \quad & \|Ax - b\|_2^2 + \lambda \|z\|_1 \\ \text{subject to} \quad & Tx - z = 0 \end{aligned}$$

The updates can be derived,

$$\begin{cases} x^{(k+1)} := (A^H A + 0.5\rho T^H T)[A^H b + 0.5\rho T^H (z^{(k)} - \mu^{(k)})] \\ z^{(k+1)} := \mathcal{T}_{\lambda/\rho}(Tx^{(k+1)} + \mu^{(k)}) \\ u^{(k+1)} := u^{(k)} + Tx^{(k+1)} - z^{(k+1)} \end{cases}$$