Journal Club #3

Sensitivity Encoding

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Outline

- **Cartesian SENSE**
  - Linear unfolding approach in image domain
- **Generalized SENSE (GSENSE)**
  - Linear approach based on iterative optimization with CG
- **Joint SENSE (JSENSE)**
  - Linear approach with joint estimation of image content and coil sensitivity based on iterative optimization
- **Self-calibrating & Temporal-regularized Radial SENSE (St. rSENSE)**
  - Linear approach similar to JSENSE
  - Self-calibrating coil sensitivities
  - Comparison with NLINV

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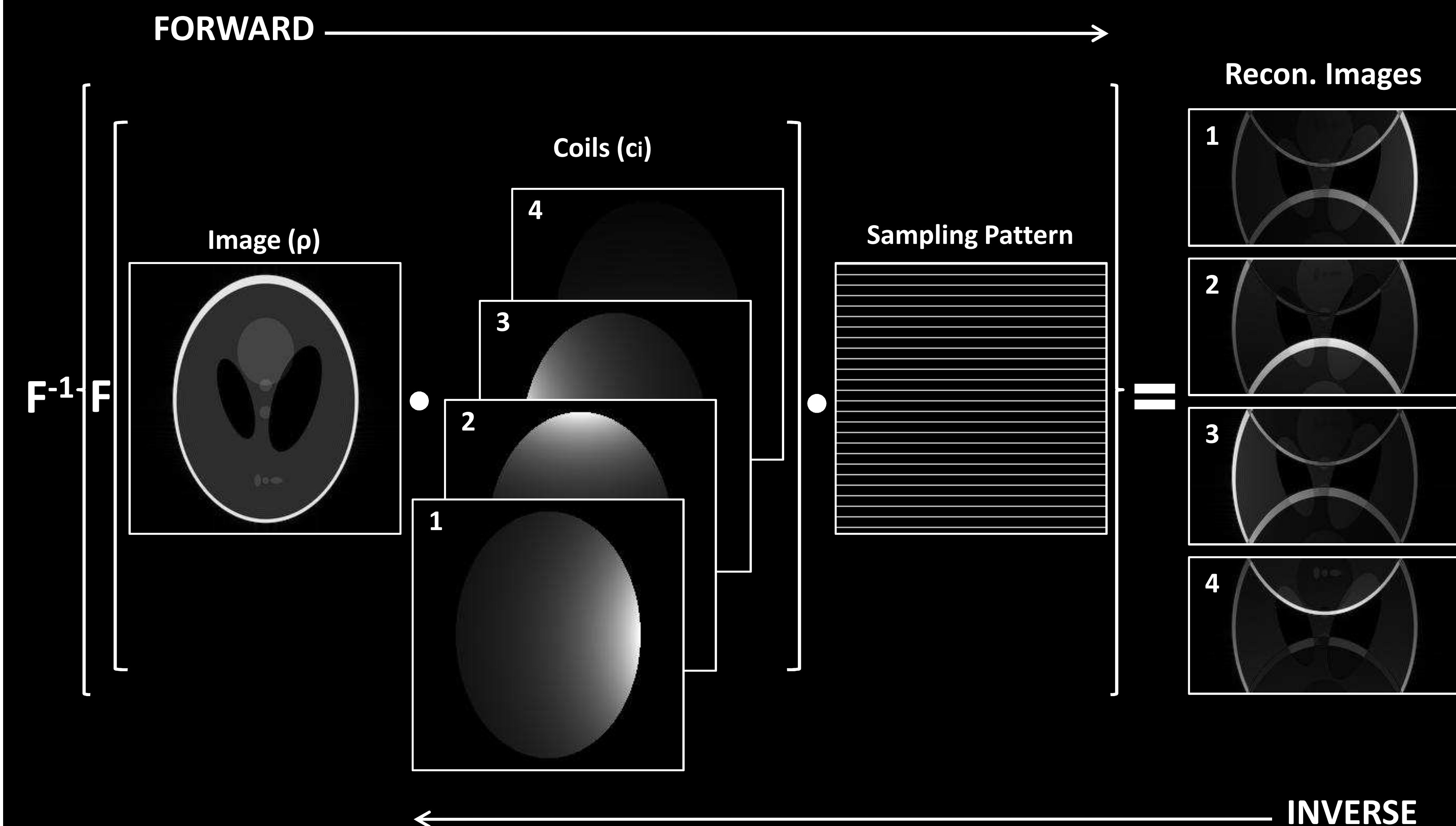
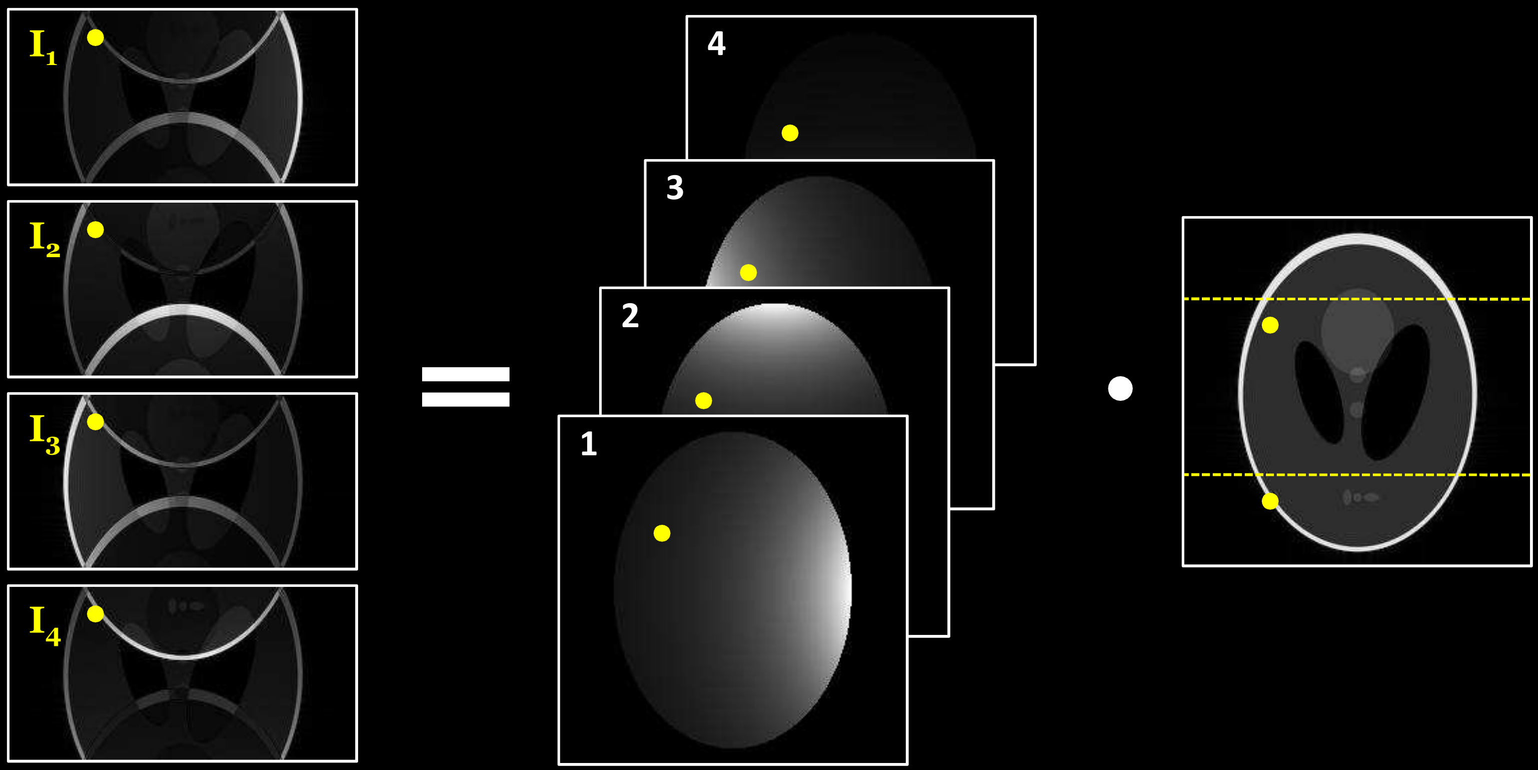
Undersampling in Cartesian Parallel MRI: Fold-in Effect



Cartesian SENSE: Unfolding in Image Domain (R = 2)



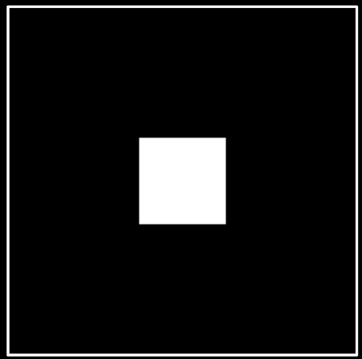
$$I_i(x,y) = C_i(x,y_1)\rho(x,y_1) + C_i(x,y_2)\rho(x,y_2) + \ldots C_i(x,y_R)\rho(x,y_R)$$

$$\vec{I} = \hat{C} \cdot \vec{\rho}$$

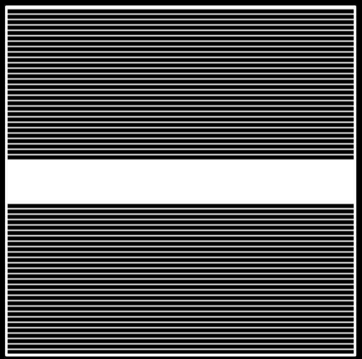
Determination of Coil Sensitivity Maps

1. **SENSE reconstruction is highly dependent on coil sensitivities**

2. **Auto-Calibration Signals (ACS) are used to calculate coil sensitivities**



Low-resolution acq.



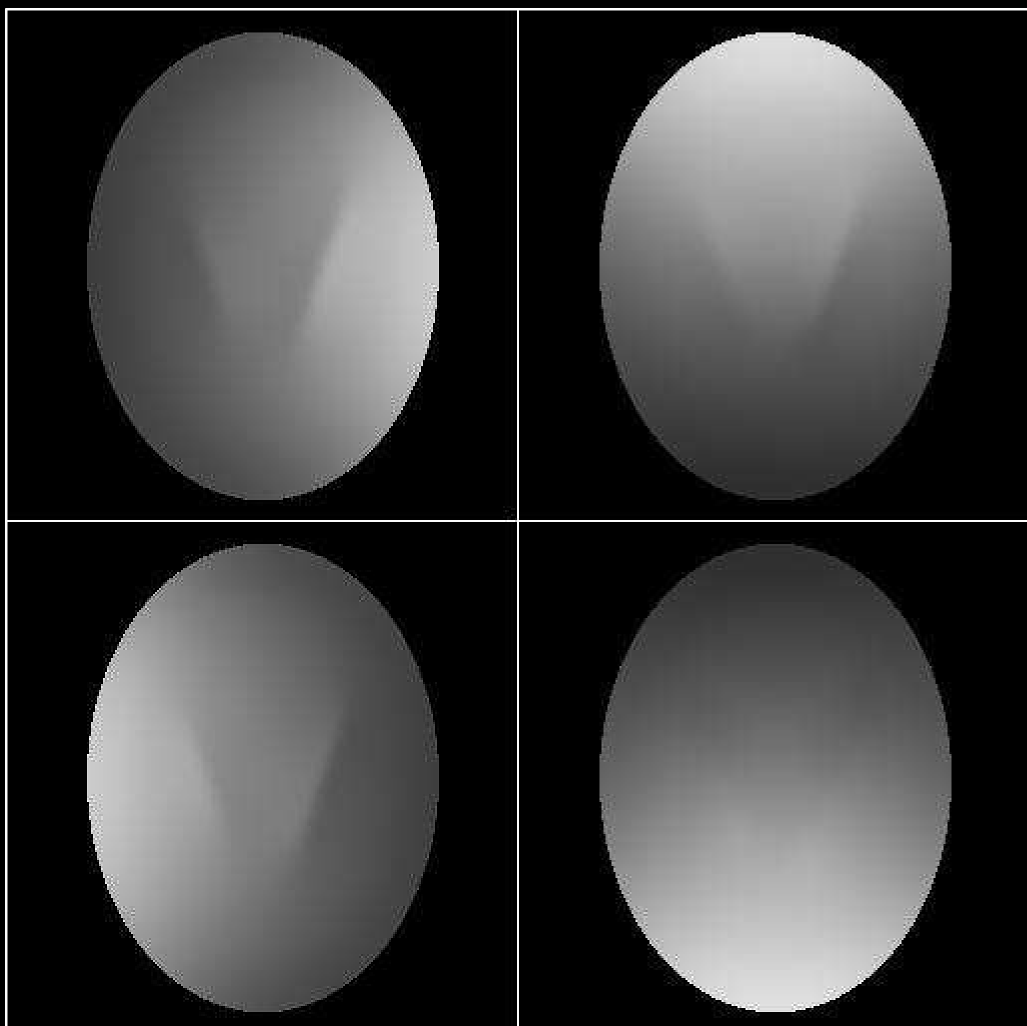
Variable-density acq.

3. **Smoothing is necessary to denoise coil sensitivity maps**
   a. Low-pass filter
   b. Polynomial fitting

Simulated Phantom Cartesian SENSE Results

Determination of Coil Sensitivities:

1. 64X64 Low-resolution acq.
2. Low-pass filter in k-space
3. Division by Sum-of-square

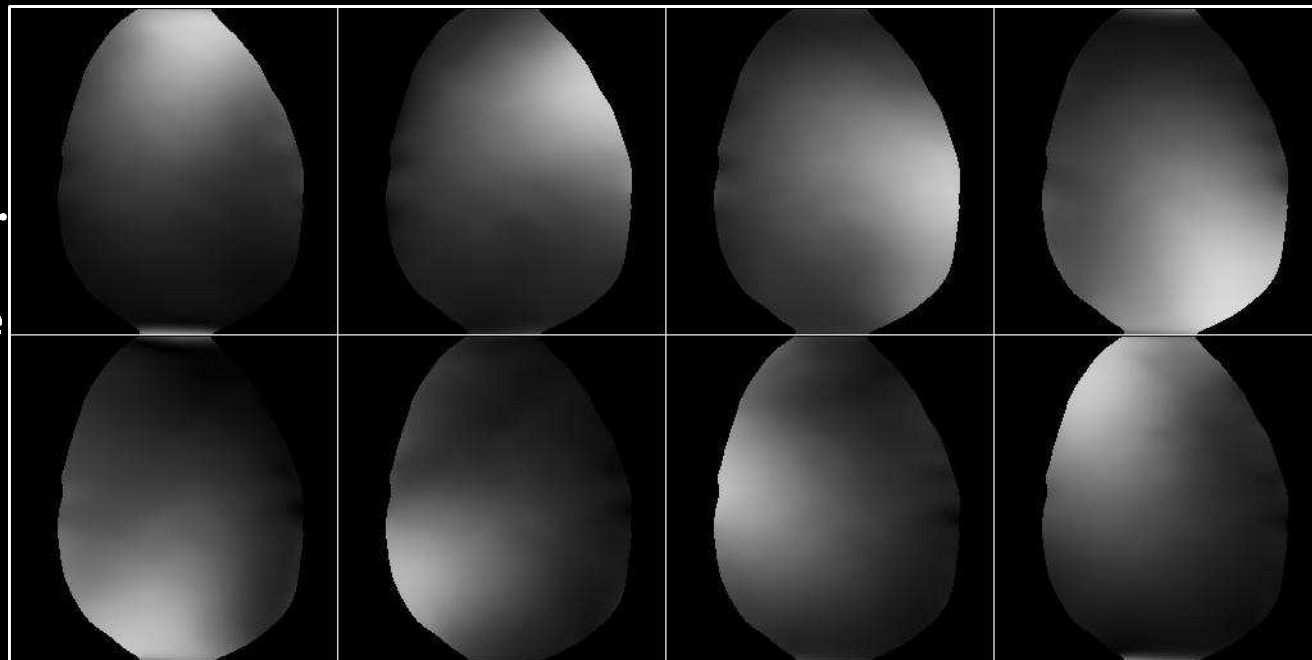


256X256 Recon.

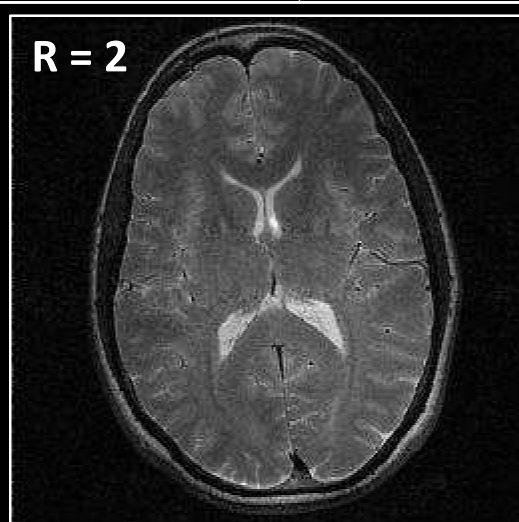
Human Brain Scan Cartesian SENSE Results

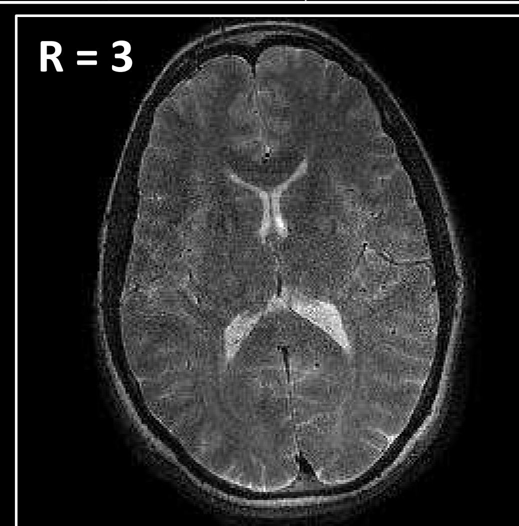
Determination of
Coil Sensitivities:

1. 64X64 Low-resolution acq.
2. Low-pass filter in k-space
3. Division by Sum-of-square



256X256 Recon.





Summary of Cartesian SENSE

1. Auto-calibration signal acquisition
2. Coil sensitivity map calculation & smoothing
3. Reconstruction: Unfolding in image domain

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Generalized SENSE (GSENSE)
Based on Conjugate Gradient

Imaging system:

$$(E^H E)\mathbf{v} = E^H \mathbf{m}$$

v: Complex pixel values of the reconstructed image
m: Complex sample values acquired

Forward operator:

$$(E\mathbf{x})_i = P \cdot F(x \cdot c_i)$$

Adjoint operator:

$$E^H \mathbf{y} = \sum_i c_i^* \cdot F^{-1}(P^H \cdot y)$$

Density & Intensity Correction in GSENSE

Density correction:

$$D = \frac{1}{d(\mathbf{k_t})}$$

$$\Rightarrow (E^H D E)\mathbf{v} = E^H D \mathbf{m}$$

Intensity correction:

$$\mathbf{I} = \frac{1}{\sqrt{\sum_i |c_i|^2}}$$

$$\Rightarrow (I E^H D E I)(I^{-1}\mathbf{v}) = I E^H D \mathbf{m}$$

Implementation of GSENSE

1. Initialization (residual):

$$\mathbf{a} = IE^H D\mathbf{m}$$

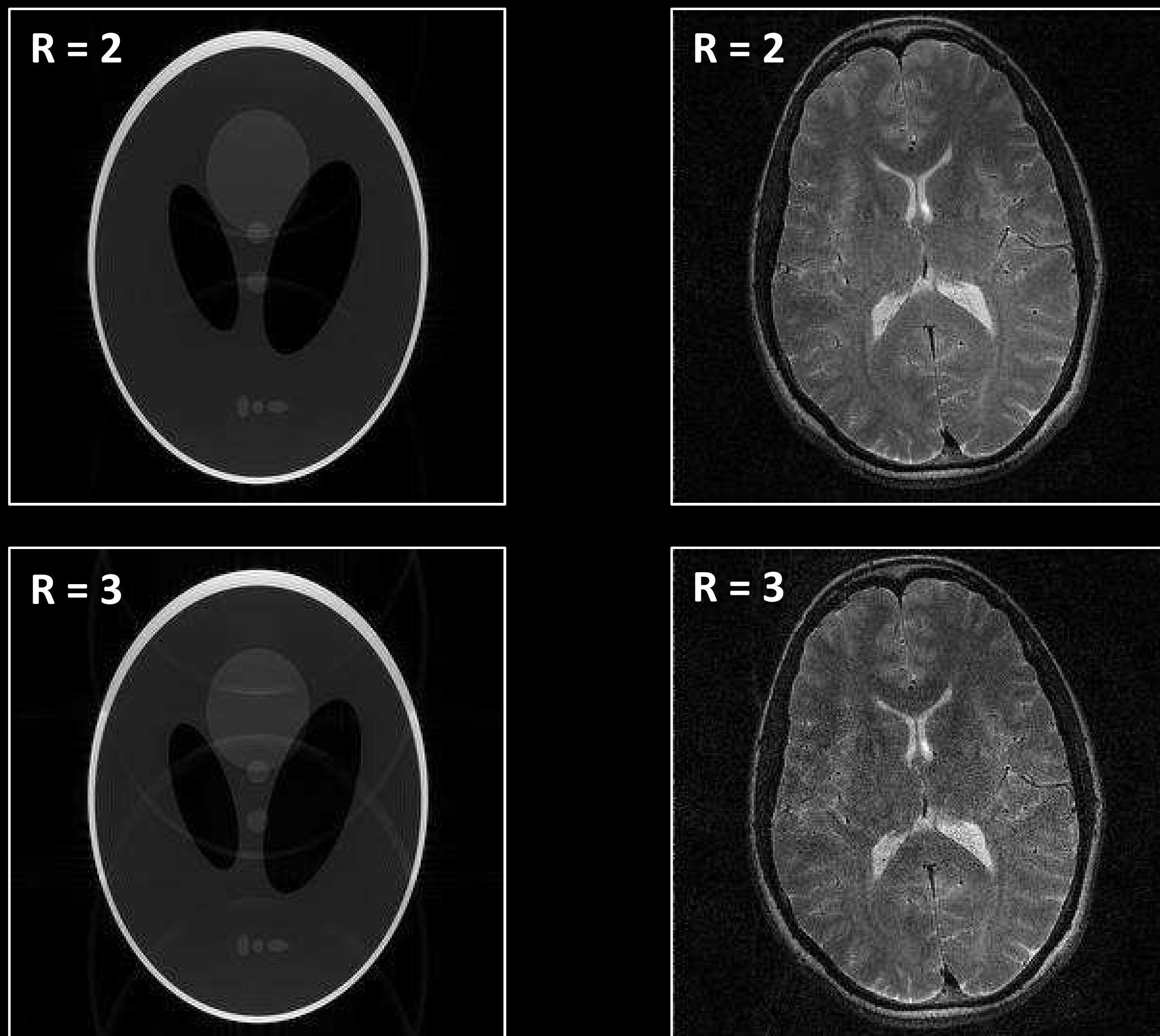
2. CG iteration to solve b:

$$(IE^H DEI)\mathbf{b} = \mathbf{a}$$

3. Approximate solution:

$$\mathbf{v}_{approx} = Ib_{approx}$$

Cartesian GSENSE Results: Low-resolution Acq.

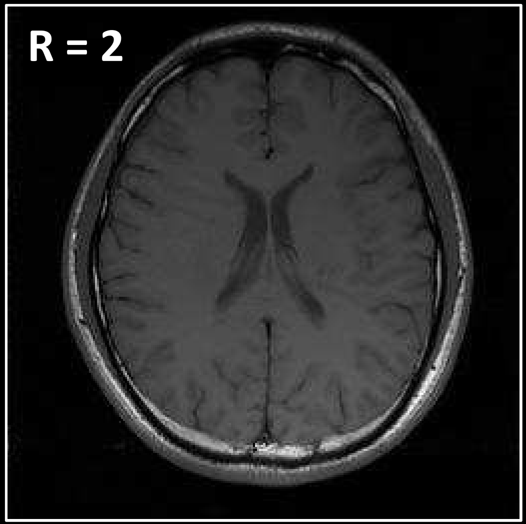


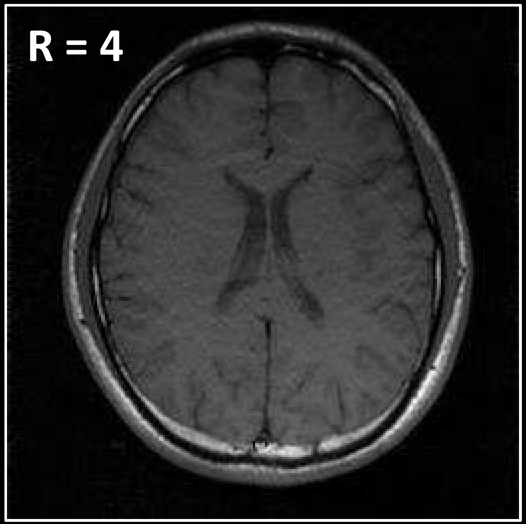
Cartesian GSENSE Results: Variable-density Acq.

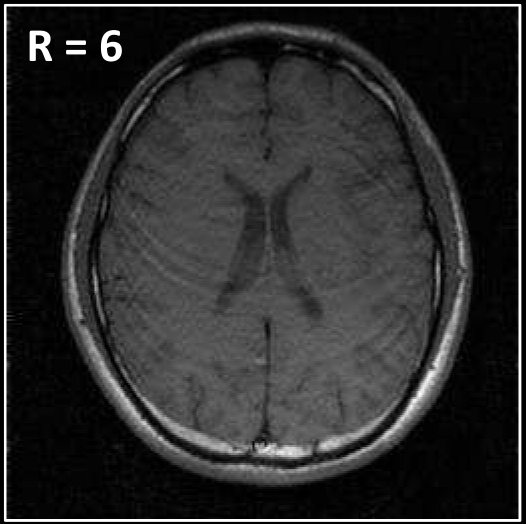
Determination of Coil Sensitivities:

1. 32X256 Variable-density acq.
2. Low-pass filter in k-space
3. Polynomial fitting
4. Division by Sum-of-square

⇨ 256X256 Recon.







Summary of GSENSE

1. Auto-calibration signal acquisition

2. Coil sensitivity map calculation & smoothing

3. System equation formulation

4. Reconstruction: Iterative conjugate gradient method

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Joint SENSE (JSENSE)

1. Auto-calibration signal acquisition
2. Coil sensitivity map calculation & smoothing
3. System equation formulation
4. Reconstruction: Iterative conjugate gradient method

(Steps 1–4: GSENSE)

5. Update coil sensitivity maps based on the reconstructed image in 4
6. Repeat steps 4 and 5 until NMSE* is low enough

* NMSE: Normalized Mean Squared Error

How to Update Coil Sensitivity Maps in JSENSE? – Least-square Solution

1. Coil sensitivity can be approximated by polynomial fitting

$$c_i(\vec{r}) = \sum_{d=0}^{D} \sum_{k+l=d} a_{i,k,l}(x-\bar{x})^k (y-\bar{y})^l$$

2. After the image is obtained from GSENSE, the system becomes

$$Fa = \mathbf{v}$$

$$F = \sum_n \rho(x_n, y_n)(x_n - \bar{x})^k (y_n - \bar{y})^l e^{-i2\pi(k_{xm}x_n + k_{ym}y_n)}$$

3. Polynomial coefficients can be calculated by pseudo-inverse

$$a = (F^H F)^{-1} F^H \mathbf{v}$$

Cartesian JSENSE Results

R = 4; 24 Auto-calibration lines; 256 x 256 Matrix; 6 JSENSE steps

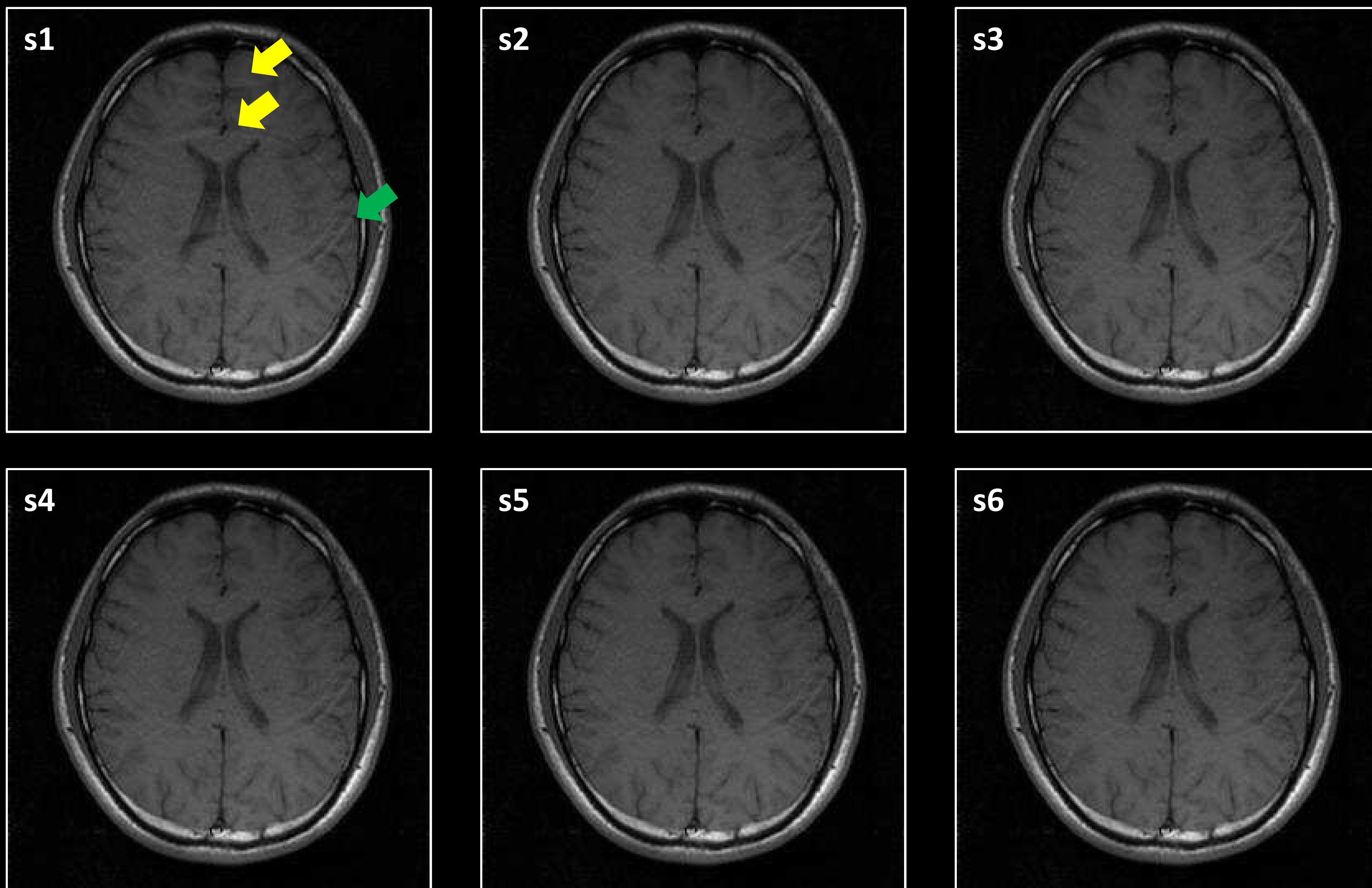
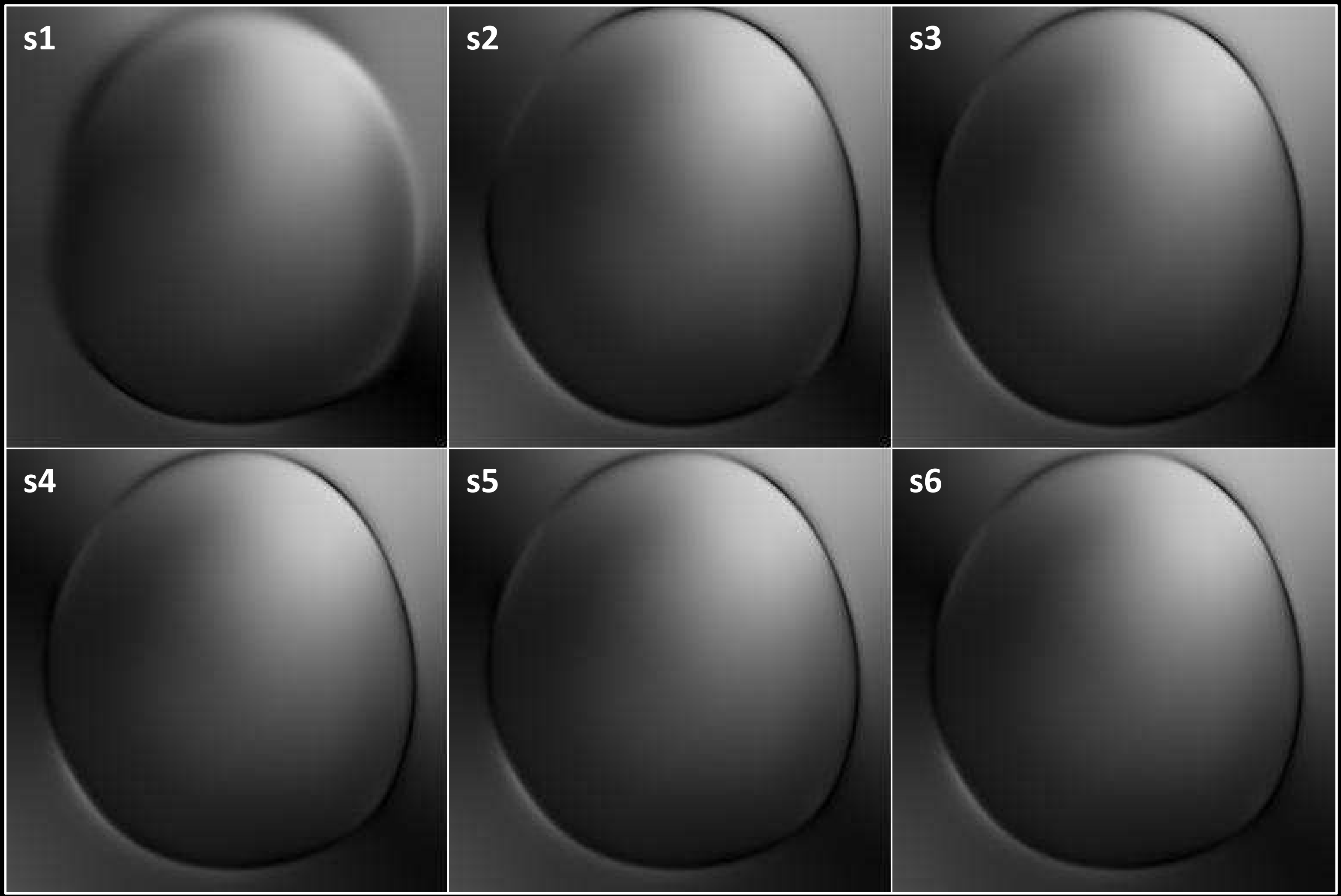


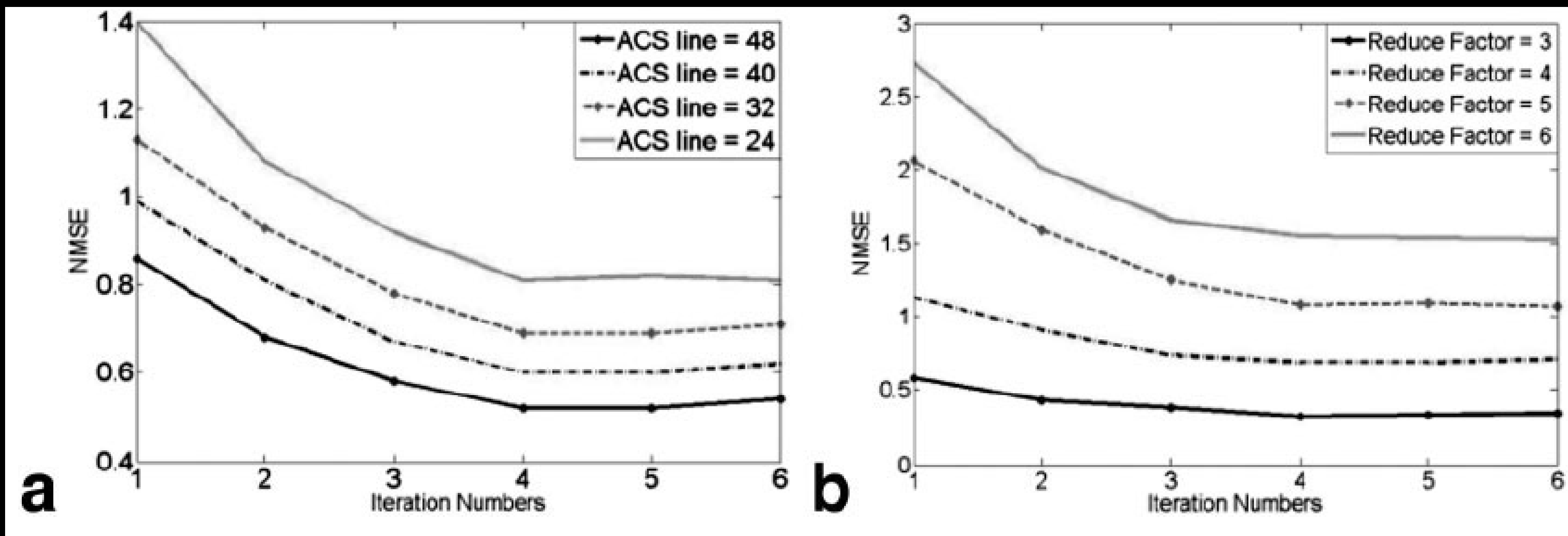
Illustration of 1st Coil Sensitivity Map in 6 JSENSE Steps

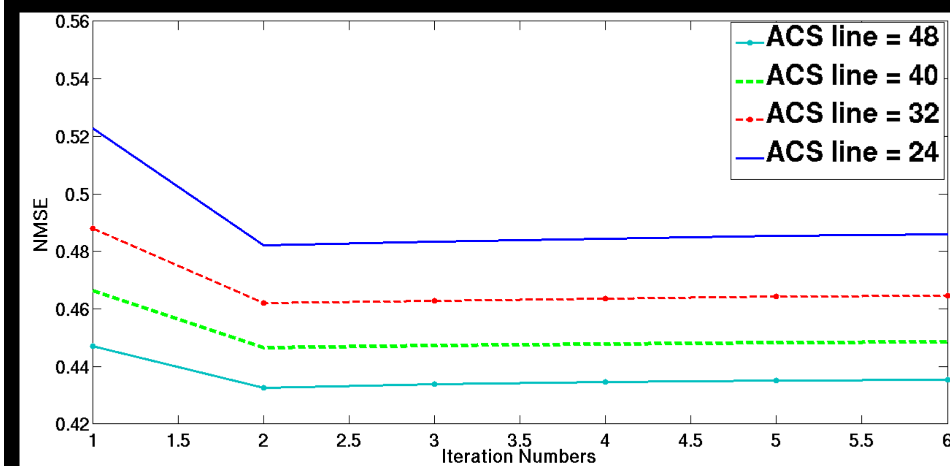


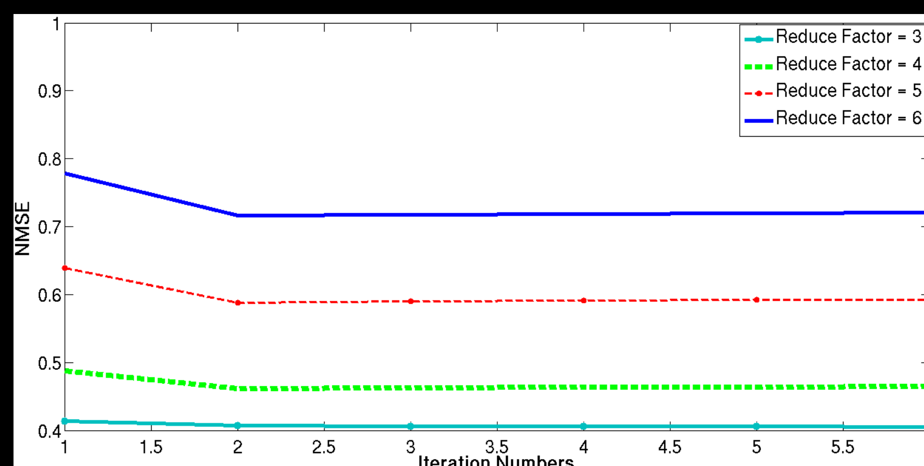
➡ JSENSE does not improve coil sensitivity maps significantly.

Quantitative Analysis of Cartesian JSENSE Results

R = 4 **32 ACS Lines**







Pros & Cons of JSENSE

Pros:

1. Divide one nonlinear problem into two linear ones and solve each one depending on the solution of another one
2. Better coil sensitivity maps estimation (less noise, smoother)
3. Truncation (Fold-in) error is reduced

Cons:

1. Still depends on auto-calibration signal for coil sensitivity maps estimation
2. Large computational complexity
3. Least-square solution of coil sensitivity coefficients does not improve coil sensitivity maps significantly

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Self-calibrating & Temporal-regularized Radial JSENSE (St. rSENSE)

1. Extract the central *k*-space area of the current frame to calibrate coil sensitivity maps

2. Iterative approach to update both image content and coil sensitivity maps (Similar to JSENSE)

3. Temporally regularize the initial guess for the next frame from the previous one

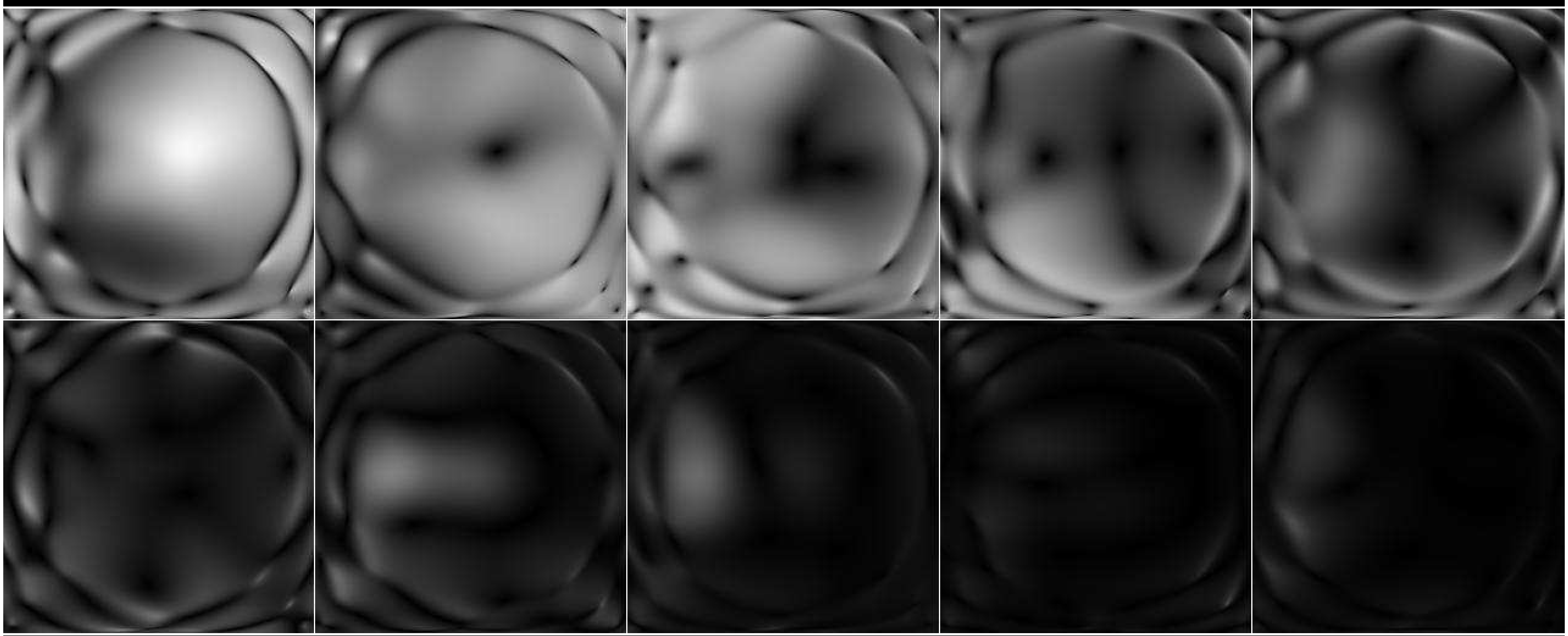
4. Repeat steps 1, 2 and 3 for the entire image series

Coil Sensitivity Maps from St. rSENSE

15 Spokes; 144 x 144 Matrix; 1.8 x 1.8 x 8 mm^3 **R = 15**

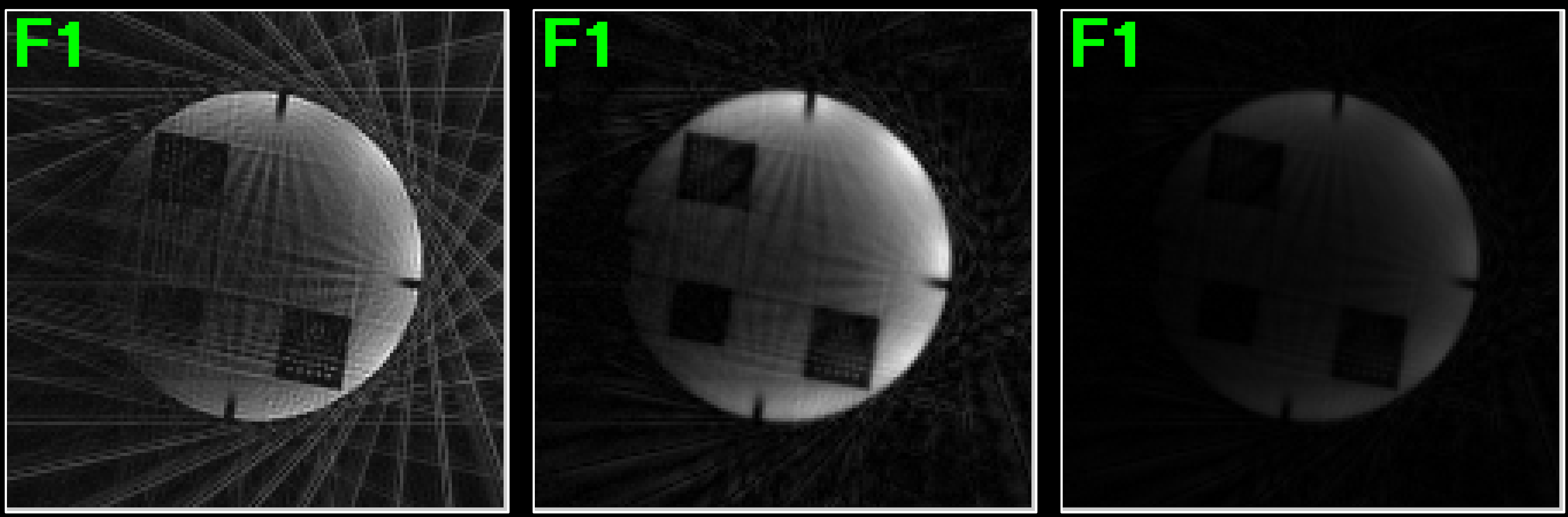
Determination of Coil Sensitivities:

1. **Grid Central fully sampled *k*-space data**
2. **Low-pass filter in *k*-space**
3. **Polynomial fitting**
4. **Division by Sum-of-square**



Importance of Temporal Regularization

15 Spokes; 144 x 144 Matrix; 1.8 x 1.8 x 8 mm^3 R = 15



Gridding & FFT; SoS

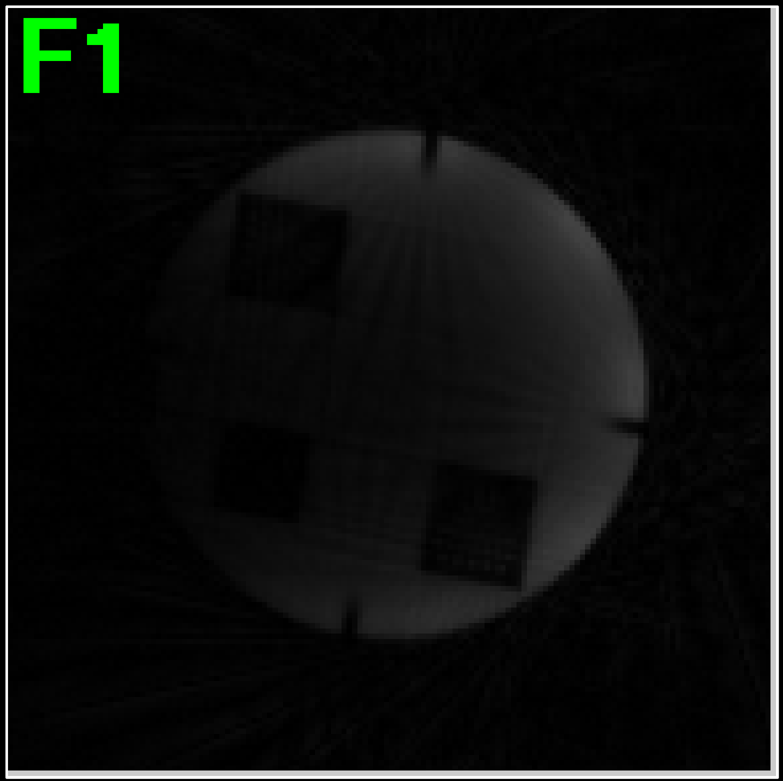
Iterative SENSE
w/o temporal regularization

St. rSENSE

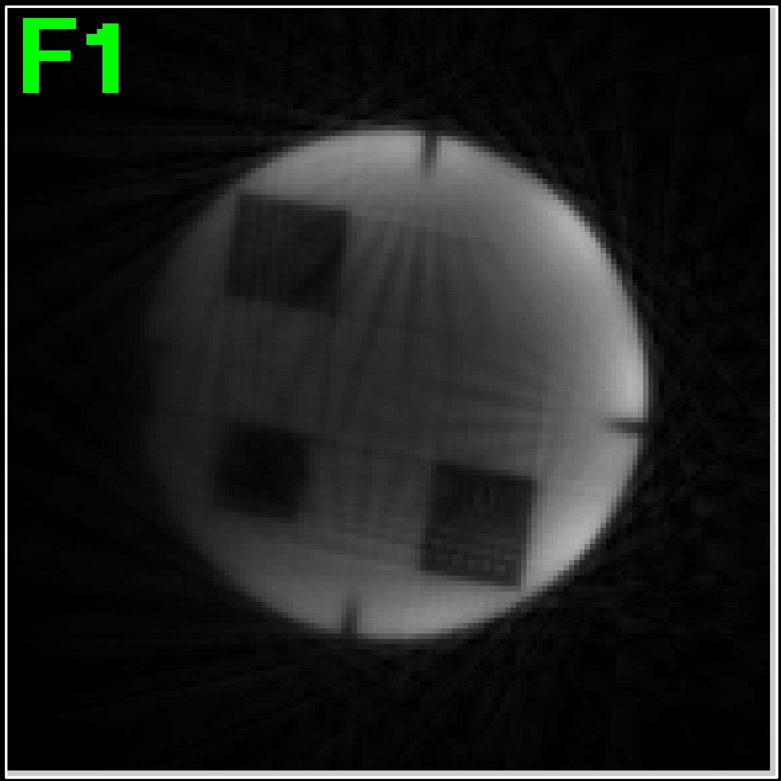
Comparison with NLINV

15 Spokes; 144 x 144 Matrix; 1.8 x 1.8 x 8 mm^3

R = 15



St. rSENSE

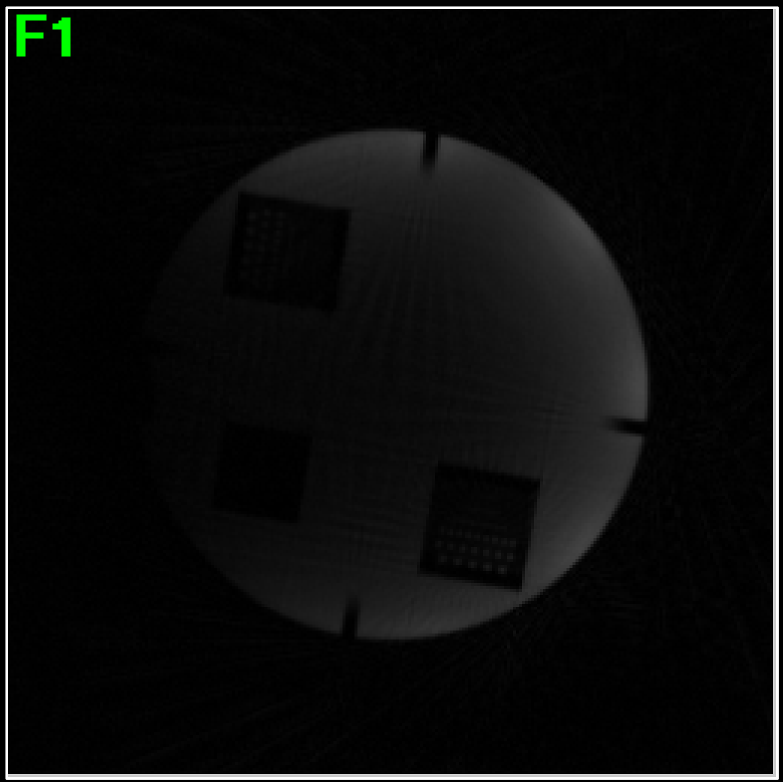


NLINV w/o median filter

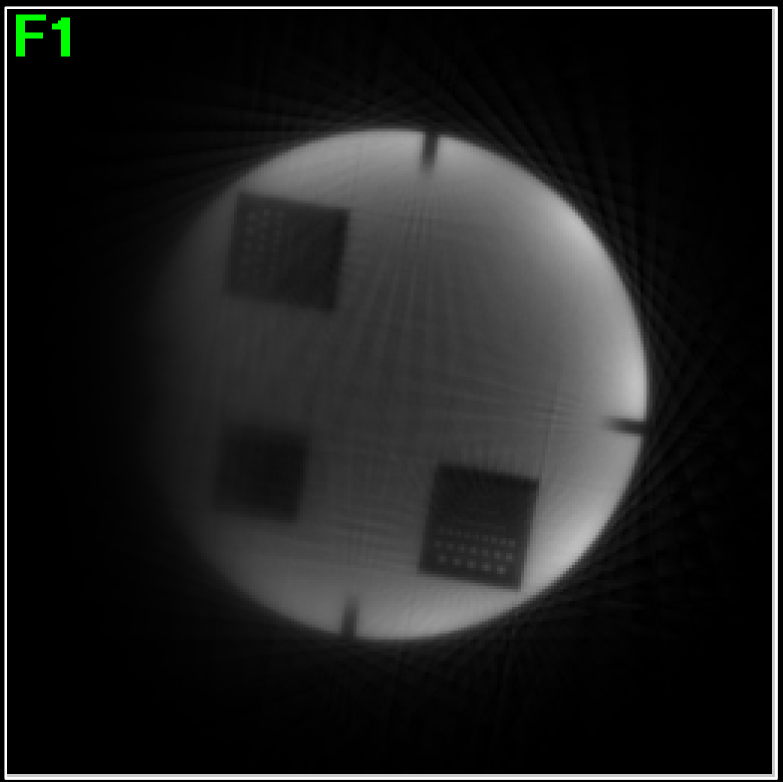
Comparison with NLINV

25 Spokes; 256 x 256 Matrix; 1.0 x 1.0 x 8 mm^3

R = 16



St. rSENSE



NLINV w/o median filter

Future Work

- Improve self-calibrating coil sensitivity calculation
- Explore appropriate temporal regularization
- Reconstruction on dynamic image series
  (motion phantom, beating hearts)

References

1. Pruessmann KP, et al, **SENSE: Sensitivity encoding for fast MRI**. *Magn Reson Med* 1999;42:952-962.

2. Pruessmann KP, et al, **Advances in sensitivity encoding with arbitrary *k*-space trajectories**. *Magn Reson Med* 2001;46:638-651.

3. Ying L, Sheng J, **Joint image reconstruction and sensitivity estimation in SENSE (JSENSE)**. *Magn Reson Med* 2007;57:1196-1202.

4. Sheng J, et al, **Improved self-calibrated spiral parallel imaging using JSENSE**. *Med Eng Phys* 2009;31:510-514.

5. Yeh EN, et al, **Inherently self-calibrating non-Cartesian parallel imaging**. *Magn Reson Med* 2005;54:1-8.

Thanks for your attention !

Questions ?
Suggestion …
Comments …

[redacted]