Unbox model-based reconstruction: Examples employing 7T diffusion MRI

Frank Z. Tan[1], Patrick A. Liebig[2], Robin M. Heidemann[2], Frederik B. Laun[3], Florian Knoll[1]
[1]Department Artificial Intelligence in Biomedical Engineering (AIBE), Friedrich-Alexander-Universität Erlangen-Nürnberg (FAU), Erlangen, Germany
[2]Siemens Healthcare GmbH, Erlangen, Germany
[3]Institute of Radiology, University Hospital Erlangen, FAU, Erlangen, Germany

Target audience: Researchers and clinicians interested in MRI data sampling & image reconstruction.

Purpose: To develop a fast-prototyping model-based MRI reconstruction framework in Python.

Methods: Model-based reconstruction [1] solves nonlinear inverse problems consisting of nonlinear forward model. For instance, the mapping from diffusion tensors to diffusion-weighted images is an nonlinear operator [2],

$$\boldsymbol{E}: s = b_0 \exp(B \times D) \quad (1)$$

where D is six-element tensors, B is the diffusion-encoding matrix based on b-values and diffusion directions, and s represents diffusion-weighted images. This nonlinear operator can be chained with parallel imaging linear operators to construct the comprehensive MR forward model,

$$\boldsymbol{A} : \boldsymbol{M}\boldsymbol{\Sigma}\boldsymbol{F}\boldsymbol{S}\boldsymbol{\Phi}\boldsymbol{E} \quad (2)$$

Here, the multi-slice diffusion-weighted images from $\boldsymbol{E}$ is multiplied by shot-to-shot phase variation ($\boldsymbol{\Phi}$) and then a set of coil sensitivity maps ($\boldsymbol{S}$). The multi-coil multi-shot multi-slice images are then Fourier transformed and subsequently phase-shifted and collapsed by the simultaneous-multi-slice (SMS) [3] operator ($\boldsymbol{\Sigma}$). Finally, the collapsed k-space is masked by its sampling pattern ($\boldsymbol{M}$). These operators are chained (composed) as $\boldsymbol{A}$ with b0 and D as input and collapsed k-space as output.

The goal in model-based reconstruction is to directly reconstruct b0 and D from acquired k-space data,

$$min_{x=(b_0, D)} \| \boldsymbol{y} - \boldsymbol{A}(x) \|_2^2 + \lambda R(x) \quad (3)$$

where the second term is regularization on the unknown parameters with regularization strength λ . This minimization problem can be rewritten in the form of alternating direction method of multipliers (ADMM) [4],

$$x^{(k+1)} = min_x \| \boldsymbol{y} - \boldsymbol{A}(x) \|_2^2 + \varrho/2 \| x - z^{(k)} + u^{(k)} \|_2^2 \quad (4.1)$$

$$z^{(k+1)} = \Gamma_{\lambda/\varrho}\left(x^{(k+1)} + u^{(k)}\right) \quad (4.2)$$

$$u^{(k+1)} = u^{(k)} + x^{(k+1)} - z^{(k+1)} \quad (4.3)$$

(4.1) is solved via IRGNM [5] employing Tikhonov regularization, while (4.2) represents singular value thresholding [6]. This solver as well as the nonlinear forward model is implemented in SigPy [7].

In vivo brain diffusion MRI with 2-shot interleaved EPI was conducted at 7 T (Terra, Siemens Healthineers, Erlangen, Germany) with 32-channel receive coils. Acquisition parameters were 1.2 mm isotropic resolution with 94 slices, 3-fold in-plane acceleration, multi-band factor 2, and total scan time of 5 minuttes. 30 diffusion directions with b-value 1000 s/mm^2 and 2 directions with b-value 50 s/mm^2.

b0 and D was initialized with 0.00001 and 0, respectively. $\lambda = 10^{-6}, \varrho = 10^{-3}$. We compared the nonlinear model-based reconstruction with reference method, where diffusion-weighted images are first reconstructed via parallel imaging and then pixel-wisely fitted to (1) via linear least squares.

Results: Figure 1 shows diffusion tensor maps from the reference method based on pixel-wise linear fitting and the proposed model-based reconstruction, respectively. Both methods exhibit noise artifacts in the central region, likely due to the strong signal decay in globus pallidus. Model-based reconstruction shows reduced noise in off-diagonal tensors (Dxy, Dxz, and Dyz).

Discussion and Conclusion: This work presents a model-based reconstruction framework in Python with both linear and nonlinear operators implemented for fast prototyping and testing of model-based reconstructions. However, model-based reconstruction with nonlinear forward models is sensitive to parameter selection and initialization.

References: **[1]** Graff C, et al. Iterative T2 estimation from highly undersampled radial fast spin-echo data. ISMRM 2006;14:925. **[2]** Basser PJ, et al. MR diffusion tensor spectroscopy and imaging. Biophys J 1994;66:259-267. **[3]** Breuer F, et al. Controlled aliasing in parallel imaging results in higher acceleration (CAIPIRINHA) for multi-slice imaging. MRM 2005;53:684-691. **[4]** Boyd S, et al. Distributed optimization and statistical learning via the alternating direction method of multipliers. Found Trends Mach Learn 2010;3:1-122. **[5]** Uecker M, et al. Image reconstruction by nonlinear inversion – Joint estimation of coil sensitivities and image content. MRM 2008;60:674-682. **[6]** Cai JF, et al. A singular value thresholding algorithm for matrix completion. SIAM J Optim 2010;20:1956-1982. **[7]** Ong F, et al. SigPy: A Python package for high performance iterative reconstruction. ISMRM 2019:4819.

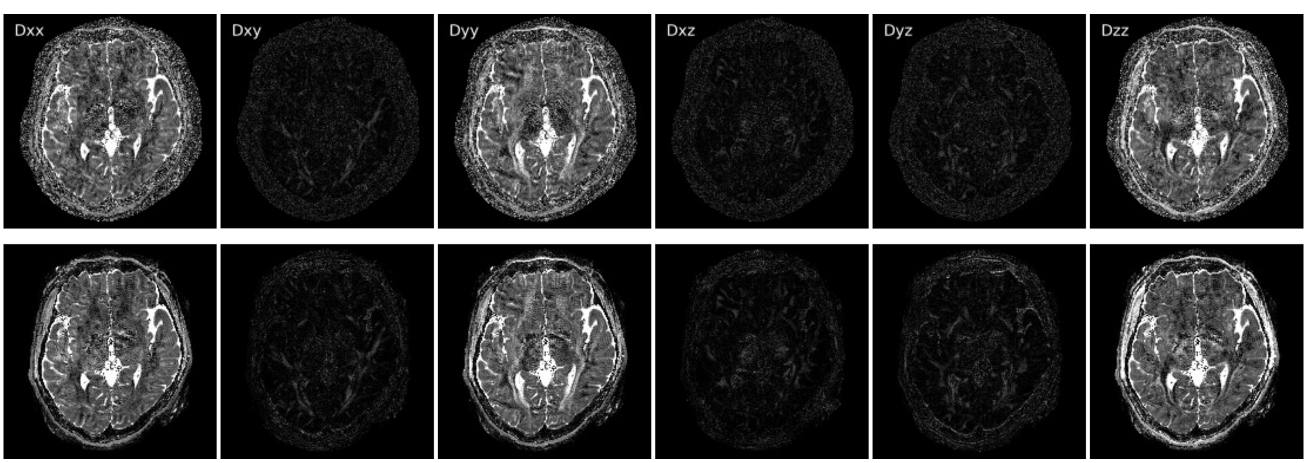

Figure 1: Diffusion tensor maps reconstructed by (top) parallel imaging with subsequent pixel-wise model fitting and (bottom) model-based reconstruction. The model-based reconstruction shows reduced noise in the off-diagonal tenor maps.